§2. 6d (2,0) SCFT's

(2,0) SCA in 6d: $Osp(8|4)$

bosonic subalgebra: $\underbrace{SO(6,2)}_{\substack{\text{conformal}\\ \text{algebra}}} \oplus Sp(2)_R$
$$Sp(2)_R = \underbrace{SO(5)}_{\text{R-symmetry}}$$

Representation is given by abelian tensor-multiplet in 6d:

- Real scalars Φ^I $(I = 1, \dots, 5)$ in 5 of $SO(5)_R$. They satisfy $\Box \Phi^I = 0$ and have $\Delta_\Phi = 2$
- Weyl fermions in 4 of $SO(5,1)$ Lorentz algebra and 4 of $SO(5)_R$ subject to symplectic Weyl reality condition:
$$\psi_{i\alpha} = \Omega_{ij} \left(C\sigma_0^T\right)_\alpha{}^\beta \psi^\dagger_{j\beta}$$
Scaling dimension: $\Delta_\psi = \frac{5}{2}$
- A real, self-dual three-form $H = *H$
$\rightarrow$ field strength of two-form gauge field B.

$\rightarrow$ $H = dB$ with $dH = d{*}H = 0$

Scaling dim: $\Delta_H = 3$

(2,0) SCFTs posses no relevant or marginal operators $\rightarrow$ no SUSY preserving deformations

String theory construction:

- Compactify type IIB string theory on ADE singularity $\mathbb{C}^2/\Gamma_{\mathfrak{g}}$ where $\mathfrak{g}$ is Lie algebra of ADE type
  $\rightarrow$ denote resulting theory by $\mathcal{T}_{\mathfrak{g}}$
  $\rightarrow$ locally characterized by a real Lie algebra $\mathfrak{g} = \bigoplus_i \mathfrak{g}_i$ where $\mathfrak{g}_i$ is either $U(1)$ or a compact, simple Lie algebra of type ADE
- $\mathfrak{g} = U(r)$ can be obtained as world-volume theory of r M5-branes in M-theory

Moduli space of vacua:

- In flat Minkowski space $\mathbb{R}^{5,1}$, $\mathcal{T}_{\mathfrak{g}}$ has moduli space of vacua:

$$\mathcal{M}_{\mathfrak{g}} = (\mathbb{R}^5)^{r_{\mathfrak{g}}} / W_{\mathfrak{g}} \,,$$

parametrized by $\langle \Phi^I \rangle$

where $r_{\mathfrak{g}}$ and $W_{\mathfrak{g}}$ are rank and Weyl group of $\mathfrak{g}$

$\rightarrow$ low-energy dynamics described by $r_{\mathfrak{g}}$ Abelian tensor multiplets valued in Cartan of $\mathfrak{g}$ "tensor branch"

$\rightarrow$ Conformal and $SO(5)_R$-symmetry are spontaneously broken

- At boundaries of moduli space: SCFT $\mathcal{T}_{\mathfrak{h}}$ with $\mathfrak{h} \subset \mathfrak{g}$ semisimple subalgebra with $r_{\mathfrak{h}} < r_{\mathfrak{g}}$ and $r_{\mathfrak{g}} - r_{\mathfrak{h}}$ ATM'

<u>The tensor branch in 6d</u>:

Restrict to breaking patterns $\mathfrak{g} \rightarrow \mathfrak{h} \oplus \mathfrak{u}(1)$

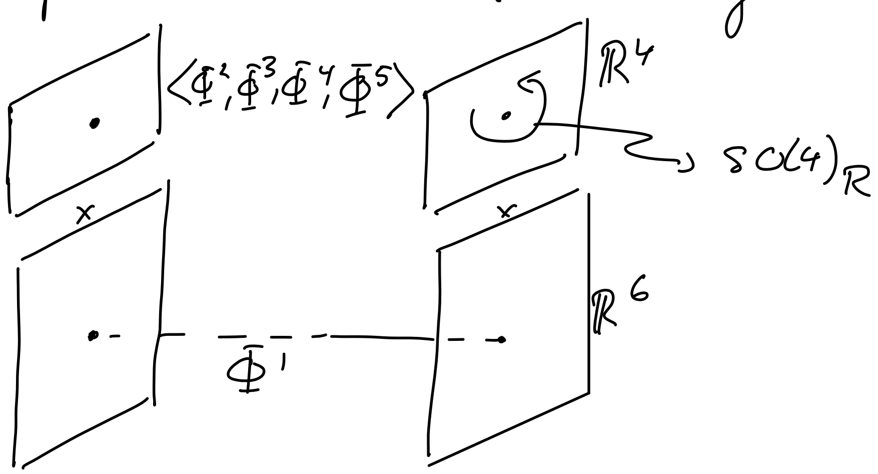
$\mathfrak{h}$ is obtained from $\mathfrak{g}$ by deleting a node in its Dynkin diagram (adjoint Higgsing)

- general properties of $\mathcal{L}_{tensor}$:

$$\mathcal{L}_{free} = -\frac{1}{2} \sum_{I=1}^{5} (\partial_\mu \Phi^I)^2 - \frac{1}{2} H \wedge * H + \text{Ferms.}$$

self-duality implies: $H \wedge * H = 0$
however, $\mathcal{L}_{free}$ formally correct

- example
consider $\mathfrak{g} = su(2)$
$\rightarrow$ adjoint Higgsing gives $su(2) \rightarrow u(1)$ by turning on scalar expectation value $\langle \Phi^1 \rangle \neq 0$ and $\langle \Phi^I \rangle = 0$ for $I \neq 1$
$\rightarrow$ R-symmetry is broken to $SO(4)_R$
interpretation in M-theory:



§2.1 Compactification to 5d:

Central assumption (motivated from string theory):

$$\text{6d } (2,0) \text{ SCFT } \mathcal{T}_{\mathfrak{g}} \xrightarrow{S^1_R} $$

S^1_R (spacial circle with radius R)

effective 5d theory below KK-scale $\frac{1}{R}$:

$\mathcal{N}=2$ SYM with gauge algebra $\mathfrak{g}$ and gauge coupling $g^2 \sim R$

effective 5d Lagrangian:

- gauge field: $A = A_\mu dx^\mu$, $\mathfrak{g}$-valued
- field strength: $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$
  $= dA - i A \wedge A$, $\mathfrak{g}$-valued
- scalars ϕ^I in 5 of $so(5)_R$, $\mathfrak{g}$-valued (R-symmetry is preserved by circle comp.)

$$\mathcal{L}_0^{(5)} = -\frac{1}{2g^2} \mathrm{Tr}_{\mathfrak{g}}\Big(F \wedge * F + \sum_{I=1}^{5} D_\mu \phi^I D_\mu \phi^I - \frac{1}{8} \sum_{I,J} [\Phi^I, \Phi^J]^2 \Big)$$
$$+ (\text{Fermions}) + (\text{higher derivatives})$$

§2.2 Compactification on 3-manifolds

Our starting point is the following configuration of M-theory fivebranes:

$$\text{spacetime:} \quad \Big\{ \overset{\text{solid torus}}{S^1 \times D^2} \text{ or } \overset{\text{lens space}}{L(k,1)_b} \Big\} \times T^* M_3 \times \mathbb{R}^2$$

$$\hphantom{\text{spacetime:} \quad \Big\{ S^1 \times D^2 \text{ or } L(k,1)_b \Big\} \times T^*} \cup$$

$$N \text{ fivebranes:} \quad \Big\{ S^1 \times D^2 \text{ or } L(k,1)_b \Big\} \times M_3$$

- Here, M_3 is an arbitrary 3-manifold, embedded in a local Calabi-Yau 3-fold $T^* M_3$ as the zero section
- $L(k,1)_b$ on the other hand is the lense space:

$$L(k,1)_b := \left\{ (z,w) \in \mathbb{C}^2 \,\middle|\, b^2 |z|^2 + b^{-2} |w|^2 = 1 \right\} / \mathbb{Z}_k$$

where $b \in \mathbb{C}$ and $\mathbb{Z}_k$ is given by

$$(z,w) \longmapsto \left(e^{2\pi i/k} z, e^{-2\pi i/k} w \right)$$

$\longrightarrow$ one can reduce the 6d (2,0)-th in two ways:

$$\text{6d } (2,0) \text{ th. on } L(k,1)_b \times M_3$$

a) $\swarrow$ 3d $\mathcal{N}=2$ th. $T[M_3]$ on $L(k,1)_b$

b) $\searrow$ complex Chern-Simons th. on M_3

Topological twisting:

The lense space can be viewed as a circle fibration over S^2:

$$\begin{array}{ccc} S^1 & \hookrightarrow & L(k,1)_b \\ & & \downarrow \\ & & S^2 \end{array}$$

$\rightarrow$ reduce the 6d (2,0) on S^1 to obtain 5d $\mathcal{N}=2$ SYM

$\rightarrow$ Lorentz and R-symmetry algebra is

$$SO(5)_L \times SO(5)_R$$

$\downarrow$ split as

$$SO(2)_L \times SO(3)_L \times SO(3)_R \times SO(2)_R$$

$SO(2)_L$: group of rotations of S^2

$SO(2)_R$: R-sym of 3d $\mathcal{N}=2$

$SO(3)_L$: Lorentz-sym. of M_3

$SO(3)_R$: rotations of cotangent bundle of M_3

$\to$ bosons and fermions of 5d $\mathcal{N}=2$ SYM transform as:

$$SO(5)_L \times SO(5)_R \longrightarrow SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$$

bosons: $(5,1) \oplus (1,5) \longrightarrow (3,1)^{(0,0)} \oplus (1,3)^{(0,0)} \oplus (1,1)^{(\pm 2,0)} \oplus (1,1)^{(0,\pm 2)}$

fermions: $(4,4) \longrightarrow (2,2)^{(\pm 1,\pm 1)}$

Implementing topological twist along M_3 amounts to replacing $SO(3)_L \cong SU(2)_L$ with the diagonal subgroup $SU(2)_L' \subset SU(2)_L \times SU(2)_R$

$\to$ under $SU(2)_L' \times U(1)_L \times U(1)_R$ the fields trf. as bosons: $(5,1)\oplus(1,5) \to 2\times 3^{(0,0)} \oplus 1^{(\pm 2,0)} \oplus 1^{(0,\pm 2)}$

(*) fermions: $(4,4) \to 3^{(\pm 1,\pm 1)} \oplus \underbrace{1^{(\pm 1,\pm 1)}}_{\text{super-charges}}$

$\to$ two copies of $3^{(0,0)}$ represent adjoint-valued one-forms on M_3

$\to$ combine into a complex gauge connection $\mathcal{A} = A + i\phi$

Reduction a):

Fivebranes wrapped on a general 3-manifold M_3 preserve 4 real supercharges (singlets in (*))

$\longrightarrow$ $\mathcal{N}=2$ in 3d

We will get back to this case later

Reduction b):

Reduction of 5d $\mathcal{N}=2$ SYM on S^2 (with suitable background fields) gives "complex Chern-Simons" theory:

$$S = \frac{q}{8\pi} \int_{M_3} \mathrm{Tr}\left(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \right)$$

$$+ \frac{\tilde{q}}{8\pi} \int_{M_3} \mathrm{Tr}\left(\bar{\mathcal{A}} \wedge d\bar{\mathcal{A}} + \frac{2}{3} \bar{\mathcal{A}} \wedge \bar{\mathcal{A}} \wedge \bar{\mathcal{A}} \right)$$

where $q = k + i\sigma$, $\tilde{q} = k - i\sigma$

$$\sigma = k \frac{1-b^2}{1+b^2}$$